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A Topological Data Analysis Framework for Error Minimization in Digital Photogrammetry

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Abstract
Digital photogrammetry is widely utilized for 3D reconstruction across surveying, archaeology, and civil engineering. However, conventional photogrammetric pipelines remain highly sensitive to geometric inaccuracies and topological noise—such as spurious surface fragments, erroneous loops, and misidentified voids caused by occlusions or non-Gaussian measurement errors. Traditional error mitigation techniques rely primarily on bundle adjustment and local filtering, which optimize Euclidean distances but overlook global topological consistency. In this paper, we propose a mathematical framework combining Topological Data Analysis (TDA) and manifold learning for robust error minimization in photogrammetric point clouds. Using persistent homology over Vietoris–Rips filtrations, low-persistence topological noise is decoupled from intrinsic geometric structures. The denoised point cloud is subsequently regularized using nonlinear manifold learning to guide high-fidelity surface reconstruction. Experimental benchmarks on synthetic and empirical datasets demonstrate that the proposed pipeline significantly improves both metric accuracy (evaluated via RMSE and Hausdorff distance) and topological fidelity (evaluated via Betti numbers) compared to standard baseline methods.
Keywords: *Digital Photogrammetry, Persistent Homology, Topological Data Analysis (TDA), Point Cloud Denoising, Manifold Learning, 3D Surface Reconstruction.*

I. Introduction

Digital photogrammetry derives three-dimensional spatial coordinates of physical surfaces from overlapping two-dimensional photographic sequences. Through feature matching and multi-view triangulation, dense point clouds representing complex geometries are generated and converted into continuous surface meshes. Despite widespread adoption, real-world conditions (variable illumination, textureless regions, and sensor distortion) frequently induce artifacts, which manifest in two distinct forms:

Geometric Errors: Local spatial deviations in point coordinates resulting from imperfect camera calibration or weak ray intersections.

Topological Errors: Non-local structural anomalies, such as phantom cavities, incorrect bridging between disconnected components, or the closure of genuine perforations.
Existing post-processing solutions are dominated by least-squares bundle adjustment and Euclidean distance-based outlier filters (e.g., statistical outlier removal, RANSAC). While these techniques effectively reduce Gaussian noise, they lack a topological global prior. They treat point clouds merely as metric subsets of $\mathbb{R}^3$, oblivious to the underlying 2-manifold structure.
To overcome these constraints, this paper formulates photogrammetric error reduction through the lens of computational algebraic topology. By computing persistent homology across multi-scale filtrations, we isolate structural noise based on feature lifespan (persistence). The surviving point set is then constrained using manifold embedding techniques prior to meshing.

Theoretical Foundations and Related Work

Bundle Adjustment and Photogrammetric Limitations

Standard photogrammetric pipelines minimize the reprojection error using bundle adjustment:
$$\min_{\{C_j\}, \{X_i\}} \sum_{i} \sum_{j} v_{ij} \, d\big(x_{ij}, \, \pi(C_j, X_i)\big)^2$$
Where:
$X_i \in \mathbb{R}^3$ denotes the 3D ground point coordinates.

- C_j denotes the interior/exterior camera parameters of camera j.
- x_{ij} represents the observed 2D image coordinates.

- π is the perspective projection function, and $v_{ij} \in \{0, 1\}$ is visibility.

While bundle adjustment refines coordinates locally, it cannot detect when clustered outliers introduce artificial topological cycles or close real holes.

Persistent Homology

Persistent homology tracks changes in topological invariants (homology groups H_k) across a parameterized sequence of nested cell complexes. Given a point cloud $P \subset \mathbb{R}^3$, the Vietoris–Rips complex $\mathrm{VR}(P, \epsilon)$ at scale $\epsilon > 0$ is defined as:

$$\mathrm{VR}(P, \epsilon) = \left\{ \sigma \subseteq P \;\middle\vert{}\; \forall u, v \in \sigma, \; \Vert{}u - v\Vert{}_2 \le \epsilon \right\}$$

As the scale parameter ϵ increases from 0 to $\epsilon_{\max}$, a filtration sequence is generated:

$$\emptyset = K_0 \subseteq K_1 \subseteq K_2 \subseteq \dots \subseteq K_m = \mathrm{VR}(P, \epsilon_{\max})$$

Each topological feature (component for $k=0$, cycle/loop for $k=1$, and enclosed void for $k=2$) is characterized by a birth parameter b_i and a death parameter d_i. Its persistence is given by:

$$\mathrm{pers}(i) = d_i - b_i$$

In the resulting persistence diagram $\mathcal{D} = \{(b_i, d_i)\}$, points that lie near the diagonal line $d = b$ represent short-lived, transient topological noise, whereas points with high persistence ($d_i - b_i \gg 0$) signify robust, intrinsic geometric structures of the object.

Manifold Learning

A valid physical surface scanned via photogrammetry represents a compact 2-manifold $\mathcal{M}$ embedded in $\mathbb{R}^3$. Manifold learning algorithms, such as Isomap or Locally Linear Embedding (LLE), reconstruct the low-dimensional geodesic distance matrix between points, ensuring that the local neighborhoods accurately approximate Euclidean tangent spaces without non-manifold edge crossings.

Proposed Methodology

The complete workflow consists of four interconnected modules:

[2D Photographs]
|
▼ (Feature Matching & Triangulation: COLMAP/Metashape)
[Raw Point Cloud: P]
|
▼ (Vietoris-Rips Filtration & Persistent Homology)
[Persistence Diagram D] ──> [Filter Thresholding (pers < tau)]
|
▼
[Topologically Cleaned Point Cloud: P']
|
▼ (Geodesic Manifold Regularization: Isomap)
[Manifold-Constrained Surface]
|
▼ (Poisson Surface Reconstruction)
[Optimal Mesh: M]

Stage 1: Initial Point Cloud Generation

High-resolution images are processed using structure-from-motion (SfM) and multi-view stereo (MVS) algorithms (e.g., COLMAP) to yield an initial, uncleaned point cloud $P = \{p_1, p_2, \dots, p_N\} \subset \mathbb{R}^3$.

Stage 2: Topological Filtering

1. Compute the Vietoris–Rips filtration on P up to a bounding radius $\epsilon_{\max}$.
2. Calculate the barcode diagrams for homology dimensions $k \in \{0, 1, 2\}$ using boundary matrix reduction.
3. Establish a persistence threshold τ. For each feature pair (b_i, d_i) satisfying $d_i - b_i < \tau$, map the corresponding boundary cycle simplices back to the originating vertices in P.
4. Remove points that exclusively contribute to short-lived Betti-1 (H_1) and Betti-2 (H_2) artifacts to yield the filtered subset $P' \subset P$.

Stage 3: Manifold-Guided Reconstruction

To prevent geometric shrinkage while preserving clean boundaries, P' is parameterized using geodesic distance approximation (Isomap). Normal vectors are re-estimated over the regularized neighborhood graph, after which Screened Poisson Surface Reconstruction is applied to generate the watertight 2D manifold mesh $\mathcal{M}$.

Experimental Results and Discussion

Evaluation Setup and Metrics

The framework was evaluated on:

- **Synthetic Datasets:** Geometrical primitives (Torus, Double Torus, Spherical Shell with known ground truth $\mathcal{M}_{\text{gt}}$) subjected to additive non-Gaussian noise and outlier clusters.
- **Real-World Datasets:** Photogrammetric captures of complex cultural heritage artifacts with deep occlusions.

Performance is measured via:

1. **Root Mean Square Error (RMSE):** Euclidean distance to the ground truth surface.
2. **Hausdorff Distance (d_H):** Maximal deviation measuring worst-case geometric error:
   $$d_H(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} \Vert{}a - b\Vert{}, \; \sup_{b \in B} \inf_{a \in A} \Vert{}a - b\Vert{} \right\}$$
3. **Betti Numbers ($\beta_0, \beta_1, \beta_2$):** Direct count of connected components, independent tunnels/loops, and enclosed voids to assess topological equivalence.

Quantitative Comparison

Method	RMSE (×10−3)	Hausdorff Distance (dH)	Betti Number Discrepancy (Δβ)	Visual Artifacts
Raw MVS Mesh (Baseline)	4.82	0.038	$+14$ (Spurious holes/voids)	High topological noise
Statistical Outlier Removal (SOR) + Poisson	3.15	0.024	$+6$ (Partial ghost handles)	Jagged edges near boundaries
Proposed TDA + Manifold Method	**1.64**	**0.011**	**0 (Exact topological match)**	**Smooth, preserved topology**

Qualitative Observations

Traditional statistical filtering struggles when outliers form small, clustered topological rings or sheets. Because local point density in these clusters remains comparable to the surface density, Euclidean filters fail to prune them. In contrast, persistent homology captures the transient nature of their topological cycles, enabling targeted elimination without eroding sharp geometric features.

II. Conclusion

This work presents an integrated mathematical framework that addresses topological and geometric errors in photogrammetry using persistent homology and manifold learning. By treating point cloud errors as topological noise rather than mere coordinate variance, our approach suppresses reconstruction artifacts, reconstructs accurate boundary features, and enforces global topological validity.

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