

A Study of The Modified Distribution Method In Modern Business

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Abstract

The modified distribution method is also known as the MODI method or the (u - v) method. This analysis provides a minimum cost solution to transportation problems. The MODI method is an efficient way to synthesize the optimum from the original feasible solution. The MODI method is an improvement of the springboard method. This presentation examines the minimization of the cost of transporting a product from multiple sources to a given destination. We are aware of the demand and sources of supply for each commodity. The purpose of the application is to expand and evaluate an integrated transportation program that meets all storage requirements with minimal transportation costs.

I. Introduction

The basic transportation problem was originally developed by F.L Hitchcock (1941) in his study entitled “The distribution of product from several sources to numerous locations. In 1947 T.C Koopmans independently published a study on “Optimum utilization of transportation system. Subsequently the linear programming formulation and the associated systematic procedure for solution were given by Gorge B. Dantzig(1951).

The Transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum. It can also be defined as to ship goods from various origins to various destinations in such a manner that the transportation cost is a minimum. The availability as well as the requirements is finite. It is assumed that the cost of shipping is linear. MODI method is an improvement over stepping stone method.

Here researcher study an important class of linear programs called the transportation model. This model studies the minimization of the cost of transporting a commodity from a number of sources to several destinations. The supply at each source and the demand at each destination are known.

II. Idea of the study

This research work represents transportation modeling approaches and forecasting techniques addressing the transportation flow of cargo containers with semi-processed goods on the selected routes from a certain number of suppliers with various production capacities to the certain points of destination. The aim is to achieve the minimum cost of transportation flow and to forecast the future for the company’s activities. Since the cost minimization directly relates to the company’s profitability of which is representing operation efficiency that can be expressed as a fraction, respective transportation modeling methods can be solved using modified distribution method. The models were studied based on a real-life data and as example of transportation. Since the forecast of future activities can be also related to the company’s strategic planning. The forecasting problem is solved by one of the most common forecasting techniques used in business life, namely the trend adjusted forecast approach.

The paper is conducted in order to introduce the transportation problem solutions by applying different methods of the transportation flow of a company, in order to find the points that could be improved and minimize transportation costs of the company. The paper was also conducted in order to show how basic figures of transportation flow can be transferred into a transportation matrix which is the basis of any transportation problem. Understanding of transportation problem methods can help to find an optimum solution for the transportation flow. Based on calculations and results of different methods and approaches to the same

transportation problem, using different cases when demand was and wasn't equal to supply were also investigated. The researcher is also looking into the forecasting problem to show how forecasting approaches can help to predict transportation activities of the company in the future. The paper studied, with the help of transportation modeling methods such as Northwest-corner, Lowest-Cost and Vogel's Approximation, using real figures and data, destinations to the terminal, terminal expenses and freight cost of transportation from the terminal of the final destination. The study investigates possible ways of minimizing the cost of transportation by using handmade calculation. The objective is to review an integral transportation schedule that meets all demands from the inventory at a minimum total transportation cost.

III. Necessary Assumption behind Transportation Problem

Let us consider a T.P involving m -origins and n -destinations. Since the sum of origin capacities equals the sum of destination requirements, a feasible solution always exists. Any feasible solution satisfying $m+n-1$ of the $m + n$ constraints is a redundant one and hence can be deleted. This also means that a feasible solution to a T.P can have at the most only $m + n - 1$ strictly positive component, otherwise the solution will degenerate.

It is always possible to assign an initial feasible solution to a T. P. in such a manner that the rim requirements are satisfied. This can be achieved either by inspection or by following some simple rules. We begin by imagining that the transportation table is blank i.e. initially all $X_{ij} = 0$. The simplest procedures for initial allocation discussed in the following section.

IV. Modified Distribution Method (MODI) or (u - v) method basic steps for solving

The modified distribution method, also known as MODI method or (u - v) method provides a minimum cost solution to the transportation problem. In the stepping stone method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with highest opportunity cost is drawn.

1. Determine an initial basic feasible solution using any one of the three methods given below:

- North West Corner Rule
- Matrix Minimum Method
- Vogel Approximation Method

2. Determine the values of dual variables, u_i and v_j , using $c_{ij} = u_i + v_j$

3. Compute the opportunity cost using $\Delta_{ij} = c_{ij} - (u_i + v_j)$.

4. Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.

5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.

6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.

8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, **add** this quantity to all the cells on the corner points of the closed path marked with plus signs, and **subtract** it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

9. Repeat the whole procedure until an optimal solution is obtained.

V. Illustration

Consider the transportation problem presented in the following table.

Distribution centre						
		D1	D2	D3	D4	Supply
Plant	P1	19	30	50	12	7
	P2	70	30	40	60	10
	P3	40	10	60	20	18
Requirement		5	8	7	15	

Determine the optimal solution of the above problem.

Solution-

An initial basic feasible solution is obtained by Matrix Minimum Method and is shown in table 1.

Table 1

Distribution centre						
		D1	D2	D3	D4	Supply
Plant	P1	19	30	50	12 ⁷	7
	P2	70 ³	30	40 ⁷	60	10
	P3	40 ²	10 ⁸	60	20 ⁸	18
Requirement		5	8	7	15	35

Initial basic feasible solution

Total Cost= 12 X 7 + 70 X 3 + 40 X 7 + 40 X 2 + 10 X 8 + 20 X 8 = **Rs. 894.**

Calculating u_i and v_j using $c_{ij} = u_i + v_j$

Substituting $u_1 = 0$, we get

$u_1 + v_4 = C_{14}$	0	+	v_4	=	12	or	v_4	=	12
$u_3 + v_4 = C_{34}$	u_3	+	12	=	20	or	u_3	=	8
$u_3 + v_2 = C_{32}$	8	+	v_2	=	10	or	v_2	=	2
$u_3 + v_1 = C_{31}$	8	+	v_1	=	40	or	v_1	=	32
$u_2 + v_1 = C_{21}$	u_2	+	32	=	70	or	u_2	=	38
$u_2 + v_3 = C_{23}$	$38 + v_3 = 40$ or $v_3 = 2$								

Table 2

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	19	30	50	12 ⁷	7	0
	P2	70 ³	30	40 ⁷	60	10	38
	P3	40 ²	10 ⁸	60	20 ⁸	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Calculating opportunity cost using $\Delta_{ij} = c_{ij} - (u_i + v_j)$.

Unoccupied cells	Opportunity cost
(P ₁ , D ₁)	$c_{11} - (u_1 + v_1) = 19 - (0 + 32) = -13$
(P ₁ , D ₂)	$c_{12} - (u_1 + v_2) = 30 - (0 + 2) = 28$
(P ₁ , D ₃)	$c_{13} - (u_1 + v_3) = 50 - (0 + 2) = 48$
(P ₂ , D ₂)	$c_{22} - (u_2 + v_2) = 30 - (38 + 2) = -10$
(P ₂ , D ₄)	$c_{14} - (u_2 + v_4) = 60 - (38 + 12) = 10$
(P ₃ , D ₃)	$c_{33} - (u_3 + v_3) = 60 - (8 + 2) = 50$

Table 3

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	-13 19	28 30	48 50	12 ⁷	7	0
	P2	70 ³	-10 30	40 ⁷	10 60	10	38
	P3	40 ²	10 ⁸	50 60	20 ⁸	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Now choose the smallest (most) negative value from opportunity cost (i.e., -13) and draw a closed path from P1D1. The following table shows the closed path.

Table 4

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	-13 19 +	28 30	48 50	-12 7	7	0
	P2	70 3	-10 30	40 7	10 60	10	38
	P3	40 2 -	10 8	50 60	20 8 +	18	8
Requirement		5	8	7	15		
v_j		32	2	2	12		

Choose the smallest value with a negative position on the closed path (i.e., 2), it indicates the number of units that can be shipped to the entering cell. Now add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

Now again calculate the values for u_i & v_j and opportunity cost. The resulting matrix is shown below.

Table 5

Distribution centre							
		D1	D2	D3	D4	Supply	u_i
Plant	P1	19 2	28 30	61 50	12 5	7	0
	P2	70 3	-23 30	40 7	-3 60	10	51
	P3	13 40	10 8	63 60	20 10	18	8
Requirement		5	8	7	15		
v_j		19	2	-11	12		

Choose the smallest (most) negative value from opportunity cost (i.e., -23). Now draw a closed path from P2D2.

Table 6

		Distribution centre				Supply	u_i
		D1	D2	D3	D4		
Plant	P1	$\begin{matrix} + \\ 19 \end{matrix} \begin{matrix} 2 \\ \end{matrix}$	$\begin{matrix} 28 \\ 30 \end{matrix}$	$\begin{matrix} 61 \\ 50 \end{matrix}$	$\begin{matrix} - \\ 12 \end{matrix} \begin{matrix} 5 \\ \end{matrix}$	7	0
	P2	$\begin{matrix} - \\ 70 \end{matrix} \begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} -23 \\ 30 \end{matrix} +$	$\begin{matrix} 40 \\ 7 \end{matrix}$	$\begin{matrix} -3 \\ 60 \end{matrix}$	10	51
	P3	$\begin{matrix} 13 \\ 40 \end{matrix}$	$\begin{matrix} - \\ 10 \end{matrix} \begin{matrix} 8 \\ \end{matrix}$	$\begin{matrix} 63 \\ 60 \end{matrix}$	$\begin{matrix} + \\ 20 \end{matrix} \begin{matrix} 10 \\ \end{matrix}$	18	8
Requirement		5	8	7	15		
v_j		19	2	-11	12		

Now again calculate the values for u_i & v_j and opportunity cost

Table 7

		Distribution centre				Supply	u_i
		D1	D2	D3	D4		
Plant	P1	$\begin{matrix} 19 \\ 5 \end{matrix}$	$\begin{matrix} 28 \\ 30 \end{matrix}$	$\begin{matrix} 38 \\ 50 \end{matrix}$	$\begin{matrix} 12 \\ 2 \end{matrix}$	7	0
	P2	$\begin{matrix} 23 \\ 70 \end{matrix}$	$\begin{matrix} 30 \\ 3 \end{matrix}$	$\begin{matrix} 40 \\ 7 \end{matrix}$	$\begin{matrix} 20 \\ 60 \end{matrix}$	10	28
	P3	$\begin{matrix} 13 \\ 40 \end{matrix}$	$\begin{matrix} 10 \\ 5 \end{matrix}$	$\begin{matrix} 40 \\ 60 \end{matrix}$	$\begin{matrix} 20 \\ 13 \end{matrix}$	18	8
Requirement		5	8	7	15		
v_j		19	2	12	12		

Since all the current opportunity costs are non-negative, this is the optimal solution. The minimum transportation cost is: $19 \times 5 + 12 \times 2 + 30 \times 3 + 40 \times 7 + 10 \times 5 + 20 \times 13 = \text{Rs. 799}$

VI. Different Cases in Transportation Problems

6.1 Degeneracy

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than $m + n - 1$ positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

1. At the initial solution
2. During the testing of the optimal solution

To resolve degeneracy, we make use of an artificial quantity (ϵ). The quantity ϵ is assigned to that unoccupied cell, which has the minimum transportation cost. For calculation purposes, the value of ϵ is assumed to be zero. The quantity ϵ is so small that it does not affect the supply and demand constraints.

6.2 Maximization in a Transportation Problem

There are certain types of transportation problems where the objective function is to be maximized instead of being minimized. These problems can be solved by converting the maximization problem into a minimization problem. The conversion can be done by subtracting each of the profit elements associated with the transportation routes from the largest profit element. The resulting values so obtained represents opportunity cost because they corresponds to the difference in profit earned by that routes and the largest profit that could be earned by any of the routes.

Forbidden roads.

Sometimes there may be situations, where it is not possible to use certain routes in a transportation problem. For example, road construction, bad road conditions, strike, unexpected floods, local traffic rules, weight or size condition etc. We can handle such type of problems in different ways:

- A very large cost represented by M or ∞ is assigned to each of such routes, which are not available.
- To block the allocation to a cell with a prohibited route, we can cross out that cell.

The problem can then be solved in its usual way.

VII. Conclusion

Understanding of transportation problem methods are help to find an optimum solution for the transportation flow. Based on calculations and results of different methods and approaches to the same transportation problem, using different cases when demand was and wasn't equal to supply. The researcher is also looking into the forecasting problem to how forecasting approaches are help to predict transportation activities of the company in the future.

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