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Coordination in Three-Level Supply Chain under Output and Demand Uncertainty

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ABSTRACT
Purpose: *The purpose of this research is to designing a contract that can coordinate three-level supply chain with output and demand uncertainty.*
Design/methodology/approach: *This paper focuses on three-level supply chain with a supplier, a manufacturer and a retailer. If the supplier and the manufacturer's production is subject to random yield and the retailer faces uncertain demand, the existence of supply chain demonstrates doubled marginalizing lead to analyses of supply chain coordination and buyback contract.*
Findings: *Although buyback contract cannot coordinate the three – level supply chain, it can improve the performance of supply chain. Therefore, we design the buyback-cost sharing contract from the perspective of risk sharing to coordination supply chain, and the profits of all members of the supply chain have been improved by Pareto. The numerical examples further to research the effects of demand and output uncertainty on supply chain performance.*
Originality/value: *Comprehensively comparing risk-sharing contract and buyback contract is only presented in the supply chain.*
Keywords: *Output uncertain; uncertain demand; supply chain coordination; risk sharing*

I. Introduction

Supply chain is a network of chain structure consists of many participants. As members of the supply chain is to pursue its own profit maximization, so the double marginalization effect cannot be avoided. In order to maximize the profit of the whole supply chain, we often need to establish appropriate contractual mechanisms to constrain and motivate members of the supply chain. Commonly used coordination contract mechanism, such as buy-back contract [1], revenue-sharing contract [2], quantity flexible contract [3] and quantity discount contract [4]. The contract for more detailed description see Cachon [5] reviewed.

In recent years, the research on the uncertainty of demand and output are mainly concerned about the two level supply chain. A large number of literatures focused on the uncertain demand, while assume the output is determined. Chen & Yano [6] has been analyzed supply chain performance and risk management under the assumption demand and weather-related circumstances. Wu [7] discussed the buy-back contract and retail price, order quantity and wholesale price. However, these papers are not able to solve the problem of coordinating in supply chain. Pasternack [8] the coordination with buy-back contract and newsboy model was studied. Buyback contract can achieve the supply chain coordination under centralized system. Cachon and Lariviere [9] are studied revenue-sharing contract and buy-back contract under demand uncertainty, the result shows that these two contracts can coordinate the supply chain; the revenue-sharing contract is equivalent to the buy-back contract. Taylor [10] for further consideration of the issue of coordination of the supply chain when sales efforts affect demand found buyback contract cannot coordinate the supply chain, in order to achieve coordination if target rebate contracts and buyback contract combinations. Khouja [11] demand uncertainty for a comprehensive review.

He & Zhang[12] propose several risk sharing contracts that distribute the random yield among parties evaluate the supply chain performances. He & Zhang[13] assume that there exists a secondary market for acquiring or disposing products by the supplier. They study both the centralized and decentralized systems. In the decentralized system, a no risk sharing contract and a risk sharing minimum commitment contract are analyzed. Güler & Keski[14] consider the revenue-sharing contract and buy-back contract on the secondary market. The results showed that both can coordinate the supply chain. Giannoccaro & Pontrandolfo [15] proposed to coordinate a three-level supply chain with revenue-sharing contract model that states the parameters satisfy certain conditions; each member's profits obtained Pareto improvement. Ding & Chen [16] demonstrated a flexible buy-back contract can coordinate the supply chain.

Unlike the above literatures, this paper studies the supplier and manufacturer output random, and retailer face a random demand, and supplier can freely determine the production inputs. Almost no one has researched the coordination problems of the three-level supply chain. We find that the supply chain has a double marginalization effect in the decentralized model, and buyback contract can weaken the double marginalization effect. Therefore, we design the buyback-cost sharing contract from the perspective of risk sharing to coordinate the supply chain.

1. Model description and assumptions

Consider the supplier - manufacturer – retailer in three-level supply chain. We assume that suppliers invest q , actual quantity of output of raw materials after the unit is uq , u interval $(0, A)(0 < A < 1)$ randomly generated variable coefficient, the distribution function and probability density function respectively are $G(u)$ and $g(u)$, means $E(u) = \mu_1$ 。Manufacturers will Q the actual production volume per unit of product is processed vQ , v interval $(0, B)(0 < B < 1)$ random variable output coefficient, the density function and probability density function respectively $H(v)$ and $h(v)$ random output variables, the mean $E(v) = \mu_2$. Market demand D is random, the probability density function and distribution function are $F(x)$ and $f(x)$, means $E(x) = \mu_x$, variance $v(x) = \delta$.

In this paper, the sequence of events presented in Figure 1: According to market demand D to determine the order quantity Q ; Manufacturers ordered from suppliers Q product; Supplier determines raw material input q . Because the supplier's output is uncertain, if the actual output is less than the number of orders $(uq < Q)$, manufacturer will buy the gap $(Q - uq)$ from the spot market at price s , which is exogenous; As manufacturers output uncertain, the actual amount of output vQ ; Retailers will sell their products to the market, the demand can be achieved。

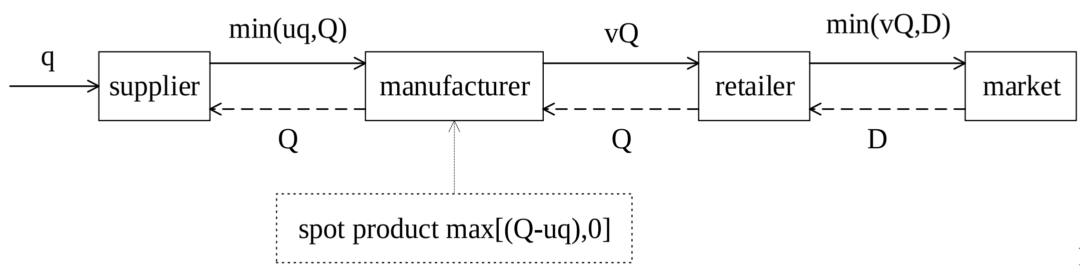

Figure. 1. Three-level supply chain schematic diagram of a random output and demand

Other symbols meanings mentioned are as follows：

p -Final market price； w_s -supplier's wholesale prices； w_m -manufacturer's wholesale price； c_s -production costs for supplier； c_m -manufacturer's processing costs； Π_i show i -expected profit, $i \in \{r, m, s\}$, three represent retailers, manufacturers and suppliers.

To facilitate further discussion of assumptions：

Hypothesis.1 $p > w_m > w_s + c_m$; **Hypothesis.2** $w_s > c_s$; **Hypothesis.3** $\mu_2 s > w_s$ 。

Hypothesis.1 and **Hypothesis.2** are ensuring that suppliers, manufacturers and retailers are willing to participate, assuming 3 in order to avoid the manufacturer does not order.

II. Model building

2.1 Focus on Supply Chain Model

Focus of supply chain with suppliers, manufacturers and retailers is designed to maximize the expected profit of the whole supply chain, namely to determine the optimal order quantity and optimal quantity of raw material inputs. The entire supply chain for the expected profit is expressed:

$$\Pi_c = E[p\min(vQ,D) - s(Q-uq)^+ - c_s q - c_m Q] = pS(Q) + sI(Q,q) - c_s q - (s+c_m)Q \quad (1)$$

The sales of market expectations

$$S(Q) = E\min(vQ,D) = \mu_2 Q - \int_0^B \int_0^{vQ} (vQ-x) f(x)h(v)dxdv$$

Expected trading volume between suppliers and manufacturers is

$$I(Q,q) = E\min(Q,uq) = Q - \int_0^{Q/q} (Q-uq)g(u)du$$

Proposition.1 Π_c is a concave function on (Q,q), the optimal (Q^c,q^c) satisfys:

$$\begin{cases} \int_0^B v\bar{F}(vQ^c)h(v)dv = \dfrac{sG(\eta^c)+c_m}{p} & (a) \\ \int_0^{\eta^c} ug(u)du = \dfrac{c_s}{s} & (b) \end{cases} \quad (2)$$

Among them, $\eta^c = Q^c / q^c$ and $\bar{F}(x) = 1 - F(x)$.

If proof is not given, then in the appendix.

Proposition.1 shows the ratio (η^c) of the optimal order quantity and optimal quantity of raw material is decided by s 、 c_s and distribution functions $g(u)$,and the formula (2) is Benchmark of supply chain coordination.

2.2 **Decentralized decision-making model**

In the decentralized decision-making model, suppliers and retailers are pursuing their own profit maximization. Expected profit of retailers、manufacturer and suppliers, respectively

$$\Pi_r = E[p\min(vQ,D) - w_m vQ] = pS(Q) - w_m \mu_2 Q \quad (3)$$

$$\Pi_m = w_m S(Q) - (w_s - s)I(Q,q) - (s+c_m)Q \quad (4)$$

$$\Pi_s = E[w_s \min(uq,Q) - c_s q] = w_s I(Q,q) - c_s q \quad (5)$$

Proposition 2. 1) Π_r is a concave function on Q ; 2) Π_s is a concave function on q ; 3) Optimal order quantity Q^w and the best raw material input q^w satisfy:

$$\begin{cases} \int_0^B v\bar{F}(vQ^w)h(v)dv = \dfrac{w_m \mu_2}{p} & (a) \\ \int_0^{Q^w/q^w} ug(u)du = \dfrac{c_s}{w_s} & (b) \end{cases} \quad (6)$$

Proposition 3. The dispersion of supply chain's optimal order quantity and optimal material inputs were less than the level of concentration of supply chain, $Q^w < Q^c$ and $q^w < q^c$.

Proposition 2 and proposition 3 shows: in a decentralized supply chain, the maximum expected profit of suppliers and retailers are existent, and the optimal order quantity and optimal raw material input satisfies the formula (6), but they were less than optimal level of concentration of the supply chain, so the presence of the supply chain double marginalization effects.

III. Coordination Mechanisms with Contract

3.1 Manufacturers to retailer's buy back contract analysis

Pasternack (2008) and Cachon (2005) noted that the buyback contract can coordinate the supply chain with demand uncertainty. Section 2 demonstrates the wholesale price contract cannot coordinate the supply chain; This section focuses on the coordination of buyback contract for analysis. In the buyback contract the retailer pays w_m for every product at the beginning of the period. The supplier pays a premium of $b(0 < b < w_m)$ to the retailer for every product that is not sold at the end of the period, then the profits of retailer, manufacturer and supplier are:

$$\Pi_r = (p-b)S(Q) - \mu_2(w_m - b)Q \quad (7)$$

$$\Pi_m = (w_m + b)S(Q) - (w_s - s)I(Q,q) - (s + c_m + \mu_2 b)Q \quad (8)$$

$$\Pi_s = w_s I(Q,q) - c_s q \quad (9)$$

Proposition 4. 1）Π_r is a concave function on Q；2）Π_s is a concave function on q；3）Optimal order quantity Q^b and the best raw material input q^b satisfy:

$$\begin{cases} \int_0^B v\overline{F}(vQ^b)h(v)dv = \dfrac{(w_m - b)\mu_2}{p-b} & (a) \\ \int_0^{Q^b/q^b} ug(u)du = \dfrac{c_s}{w_s} & (b) \end{cases} \quad (10)$$

Proposition 5. 1) The optimal order quantity and optimal raw material input quantity under buyback contract are greater than the dispersion Supply Chain，that is $Q^w < Q^b$ and $q^w < q^b$；2）The ratio of the optimal order quantity and optimal raw material input quantity under buyback contract is greater than the centralized supply chain's ratio, that is $\eta^b > \eta^c (\eta^b = Q^b/q^b)$。

Proposition 5 implies that the buyback contract weakens the double marginalization effect of the supply chain. However, the optimal order quantity and optimal raw material input cannot reach the level of centralization of supply chain, so the supply chain cannot be coordinated.

3.2 Supply chain coordination based on buyback-cost sharing contract

The 3.1 section analysis the buy-back contract cannot coordinate the three-level supply chains with output and demand uncertainty, which is due to a lack of risk sharing of output uncertainty. Therefore, we design the buyback-cost sharing contract (b, k, w_s, w_m) from the perspective of risk sharing. Among them, $k(k < c_s)$ represents the raw materials cost subsidies that manufacturer gives to supplier. The meaning of the other parameters as above. Then the retailer, manufacturer and supplier's expected profit are:

$$\Pi_r = (p-b)S(Q) - \mu_2(w_m - b)Q \quad (11)$$

$$\Pi_m = (w_m + b)S(Q) - (w_s - s)I(Q,q) - (s + c_m + \mu_2 b)Q - kq \quad (12)$$

$$\Pi_s = w_s I(Q,q) - (c_s - k)q \quad (13)$$

Proposition 6. In the buyback-cost sharing contract, 1）Π_r is a concave function on Q；2）Π_s is a concave function on q；3）Optimal order quantity Q^s and raw material input q^s satisfy:

$$\begin{cases} \int_0^B v\overline{F}(vQ^s)h(v)dv = \dfrac{\mu_2(w_m - b)}{p-b} \\ \int_0^{\eta^s} ug(u)du = \dfrac{c_s - k}{w_s} \end{cases} \quad (14)$$

Proposition 7. If the buyback-cost sharing contract (b, k, w_s, w_m) parameters satisfy

$$\begin{cases} b = \dfrac{p[\mu_2 w_m - sG(\eta^c) - c_m]}{\mu_2 p - sG(\eta^c) - c_m} \\ k = \dfrac{(s - w_s)c_s}{s} \end{cases} \quad (15)$$

We got $Q^s = Q^c$ and $q^s = q^c$, then the supply chain achieves coordination;
Retailer, manufacturer and supplier's profits are

$$\begin{cases} \Pi_r^c = \dfrac{p-b}{p}\Pi_c \\ \Pi_m^c = \dfrac{b}{p}\Pi_c - w_s\bar{G}(\eta^c)Q^c \\ \Pi_s^c = w_s\bar{G}(\eta^c)Q^c \end{cases} \quad (16)$$

Proposition 7 illustrates the buyback-cost sharing contract is not only can coordinate the three-level supply chain, but also retailer, manufacturer and supplier can arbitrarily distribute the profits of whole supply chain.

Proposition 9. If the buyback-cost sharing contract (b, k, w_s, w_m) parameters satisfy

$$\begin{cases} \max\{(w_s + c_m), \dfrac{[\Pi_m^* + w_s\bar{G}(\eta^c)Q^c][\mu_2 p - sG(\eta^c) - c_m]}{\Pi_c^*\mu_2} + \dfrac{sG(\eta^c) + c_m}{\mu_2}\} \\ < w_m < \min\{\dfrac{\mu_2 p\Pi_c^* - [\mu_2 p - sG(\eta^c) - c_m]\Pi_r^*}{\Pi_c^*\mu_2}, p\} \\ \max\{\dfrac{\Pi_s^*}{Q^c G(\eta^c)}, c_s\} < w_s < \mu_2 s \end{cases} \quad (17)$$

Then the profits of all members of the supply chain have been improved by Pareto.

Proof Compared to the wholesale price contract, the condition of retailer, manufacturer and supplier to accepting buyback contract is $\Pi_r^c > \Pi_r^*, \Pi_m^c > \Pi_m^*, \Pi_s^c > \Pi_s^*$. Combined with the formula (3)-(6), (16), hypotheses 1 and 3 can be obtained formula (17).

IV. Example analysis

In order to gain more insights, this section presents the following numerical analysis. Suppose the supplier's output is subject to $u \sim U(0,1)$ uniform distribution, the manufacturer's output is subject to $v \sim (0.4,1)$ uniform distribution. Demand is subject to $D \sim N(100,10)$ normal distribution, other parameters: $p = 1.5$, $w_m = 1.0$, $c_m = c_s = 0.5$, $s = 0.8$.

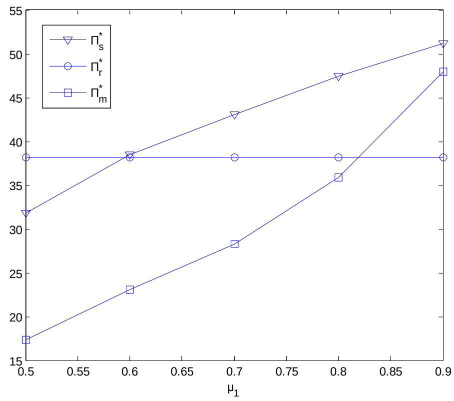

Figure. 2. Expected profit of supply chain versus μ_1

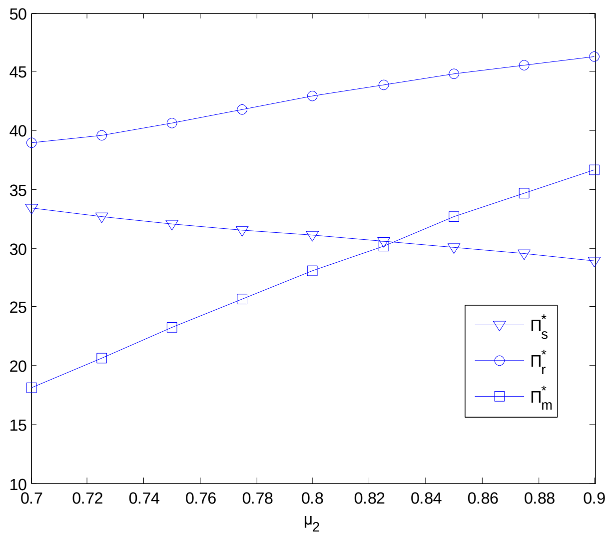

Figure. 3 Expected profit of supply chain versus μ_2

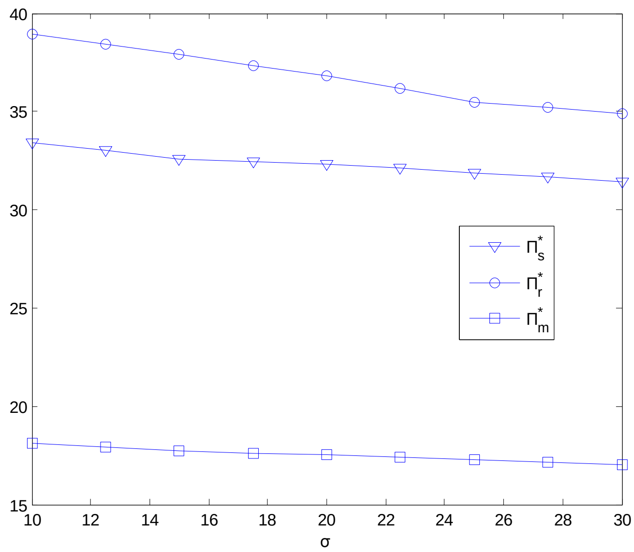

Figure. 4. Expected profit of supply chain versus σ

From figure 2 to figure 4, we can know: 1) the profit of the manufacturer and supplier are increasing with the increase of the supplier's expected output, and retailer's profit unchanged. 2) the profit of the retailer and manufacturer are increasing with the increase of the manufacturer's expected output, but the supplier is the opposite. 3) the profit of the retailer, manufacturer and supplier are descending with the increase of the variance.

V. Summary

This paper studies the coordination of three-level supply chain, which includes a retailer, a manufacturer and a supplier. At the same time, there is uncertainty about the output of the manufacturer and the supplier. The centralized model and decentralized model of supply chain are analyzed. We find that the supply chain has a double marginalization effect in the decentralized model. Next, the model of buyback contract is put forward. Although the buyback contract can weaken the double marginalization effect, it also can't coordinate the supply chain. Therefore, we design the buyback-cost sharing contract from the perspective of risk sharing, and proving the validity of the contract to coordinate the supply chain. In addition, improving the expected output and the accuracy of demand forecast can increase the expected profit of the supply chain.

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Proof for proposition 1

From formula (1) we get

$\partial^2\Pi_c / \partial^2 Q = -[p\int_0^B v^2 h(v) f(vQ)dv + sg(Q/q)/q] < 0$, $\partial^2\Pi_c/\partial^2 q = -sQ^2 g(Q/q)/q^3 < 0$,

$\partial^2\Pi_c/\partial Q\partial q = sQg(Q/q)/q^2$,

$\partial^2\Pi_c/\partial^2 Q \times \partial^2\Pi_c/\partial^2 q - \{\partial^2\Pi_c/\partial Q\partial q\}^2 = g(Q/q)psQ^2\int_0^B v^2 h(v)F(vQ)dv/q^3 > 0$.

Therefore, the Hessian matrix

$H(q,Q) = \begin{pmatrix} \frac{\partial^2\Pi_c}{\partial Q^2} & \frac{\partial^2\Pi_c}{\partial q\partial Q} \\ \frac{\partial^2\Pi_c}{\partial Q\partial q} & \frac{\partial^2\Pi_c}{\partial q^2} \end{pmatrix}$ is a negative definite matrix, so Π_c is a concave function on (Q,q), making $\partial\Pi_c/\partial Q = 0$ and $\partial\Pi_c/\partial q = 0$, we can get the formula (2).

Proof for proposition 2 and 3

1）From the formula（3）and formula（5）,we have $\partial^2\Pi_r/\partial Q^2 = -p\int_0^B v^2 h(v) f(vQ)dv < 0$, $\partial^2\Pi_s/\partial q^2 = -w_s Q^2 g(Q/q)/q^3 < 0$, therefore Π_r and Π_s are a concave function on Q and q , respectively. Ordering $\partial\Pi_r/\partial Q = 0$ and $\partial\Pi_s/\partial q = 0$, we can get the formula (6)；2）From the formula (17) , we know that $w_m\mu_2/p > [sG(\eta^c) + c_m]/p$, combined formula（2a）and（6a）get $Q^w < Q^c$, because $s > w_s$ get $c_s/s < c_s/w_s$, combined formula（2a）and（6a）get $Q^w/q^w > \eta^c$, $q^w < q^c$ 。

Proof for proposition 5

Because $s > w_s$ ， combined （2b）、（6b） and （10b） get $\eta^b = Q^b \big/ q^b = Q^w \big/ q^w > \eta^c$ ， because $(w_m \mu_2 - b\mu_2) \big/ (p-b) < w_m \mu_2 \big/ p$ ， integral（6a）and（10a）get $Q^b > Q^w$ ， so $q^b > q^w$ 。

Proof for proposition 6

1) The equation (15) generation into the equation (14),we have $\dfrac{\mu_2 (w_m - b)}{p-b} = \dfrac{\mu_2 w_m}{p}$ 、 $\dfrac{c_s - k}{w_s} = \dfrac{c_s}{s}$.

Combination with the equation (2), we known $Q^s = Q^c$ and $q^s = q^c$ ， so the buyback-cost sharing contract can coordinate the supply chain; 2) the equation (15) generation into the equation (11) and (13), combined with the equation (2), we have

$$\Pi_r^c = (p-b)\{pS(Q^c) - [sG(\eta^c) + c_m]Q^c\} \big/ p = (p-b)\Pi_c^* \big/ p$$

$$\Pi_s^c = w_s \overline{G}(\eta^c) Q^c$$

$$\Pi_m^c = \Pi_c^* - \Pi_s^c - \Pi_r^c = b\Pi_c \big/ p - w_s \overline{G}(\eta^c) Q^c$$